LA-UR-14-21554

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Title:	Reevaluation of the Jezebel Benchmark
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Intended for:	University of New Mexico, Department of Chemical and Nuclear Engineering, seminar
Issued:	2014-03-10

Reevaluation of the Jezebel Benchmark

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March 11, 2014

Abstract (not a viewgraph)

Every nuclear engineering student is familiar with Jezebel, the homogeneous bare sphere of plutonium first assembled at Los Alamos in 1954-1955. The actual Jezebel assembly was neither homogeneous, nor bare, nor spherical; nor was it singular – there were hundreds of Jezebel configurations assembled. The Jezebel benchmark has been reevaluated for the International Criticality Safety Benchmark Evaluation Project (ICSBEP) Handbook. Logbooks, original drawings, mass accountability statements, internal reports, and published reports have been used to model four actual three-dimensional Jezebel assemblies with high fidelity.

Because the documentation available today is often inconsistent, three major assumptions were made regarding plutonium part masses and dimensions. The first was that the assembly masses given in Los Alamos report LA-4208 (1969) were correct, and the second was that the original drawing dimension for the polar height of a certain major part was correct. The third assumption was that a change notice indicated on the original drawing was not actually implemented. This talk will describe these assumptions, the alternatives, and the implications.

Since the publication of the 2013 ICSBEP Handbook, the actual masses of the major components have turned up. Our assumption regarding the assembly masses was proven correct, but we had the mass distribution incorrect. Work to incorporate the new information is ongoing, and this talk will describe the latest assessment.

Jezebel

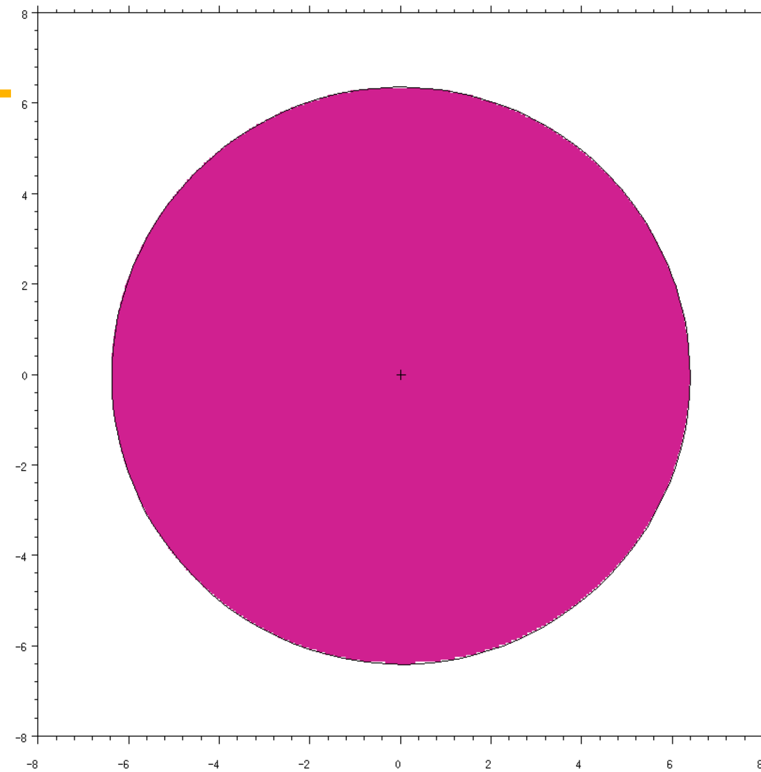
- Jezebel is a one-dimensional homogeneous bare sphere critical plutonium benchmark.

  + Radius 6.3849 cm

  + Density 15.61 g/cm^3

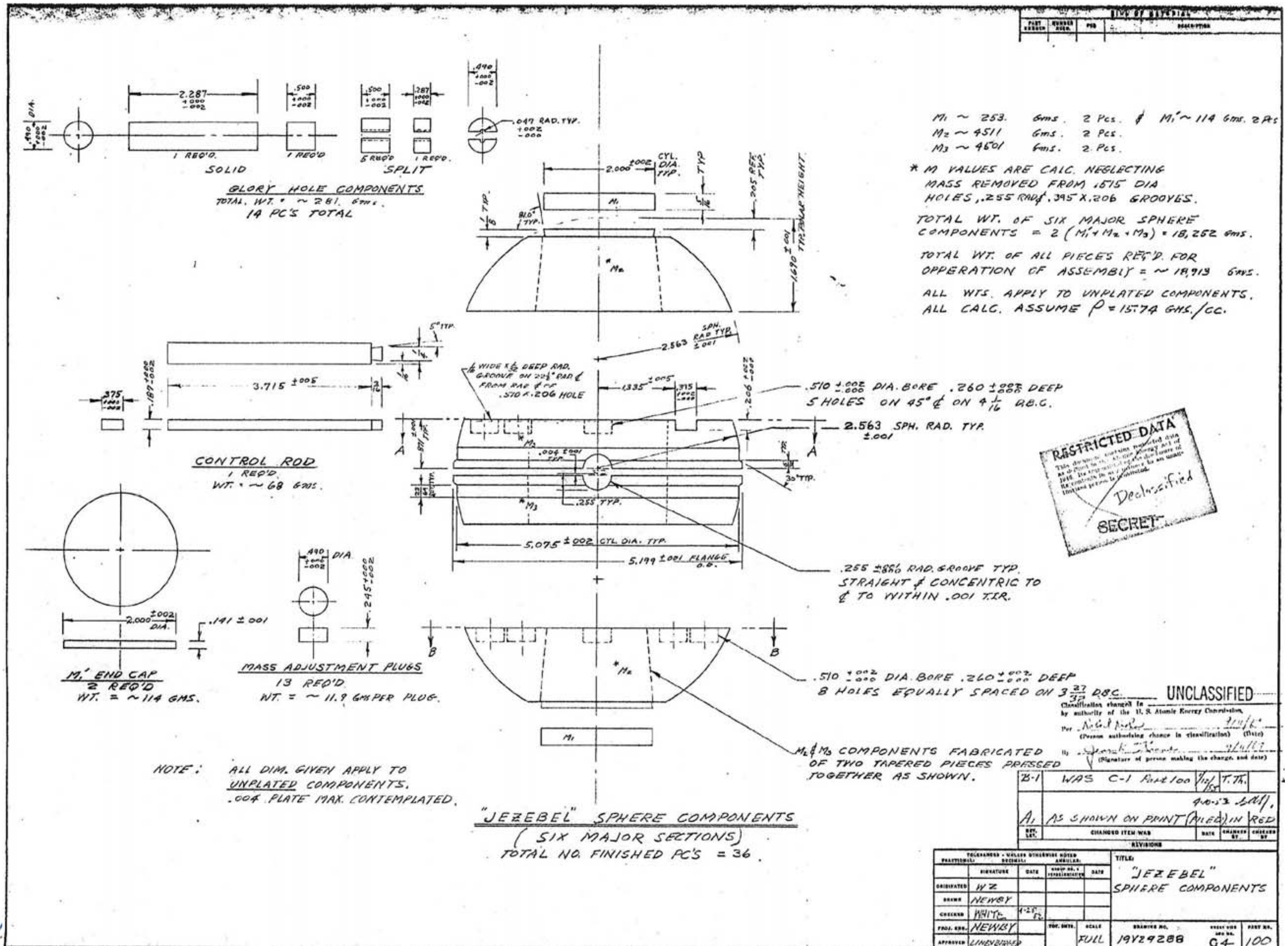
  + Mass 17,020 ± 100 g Pu alloy

  + Material:



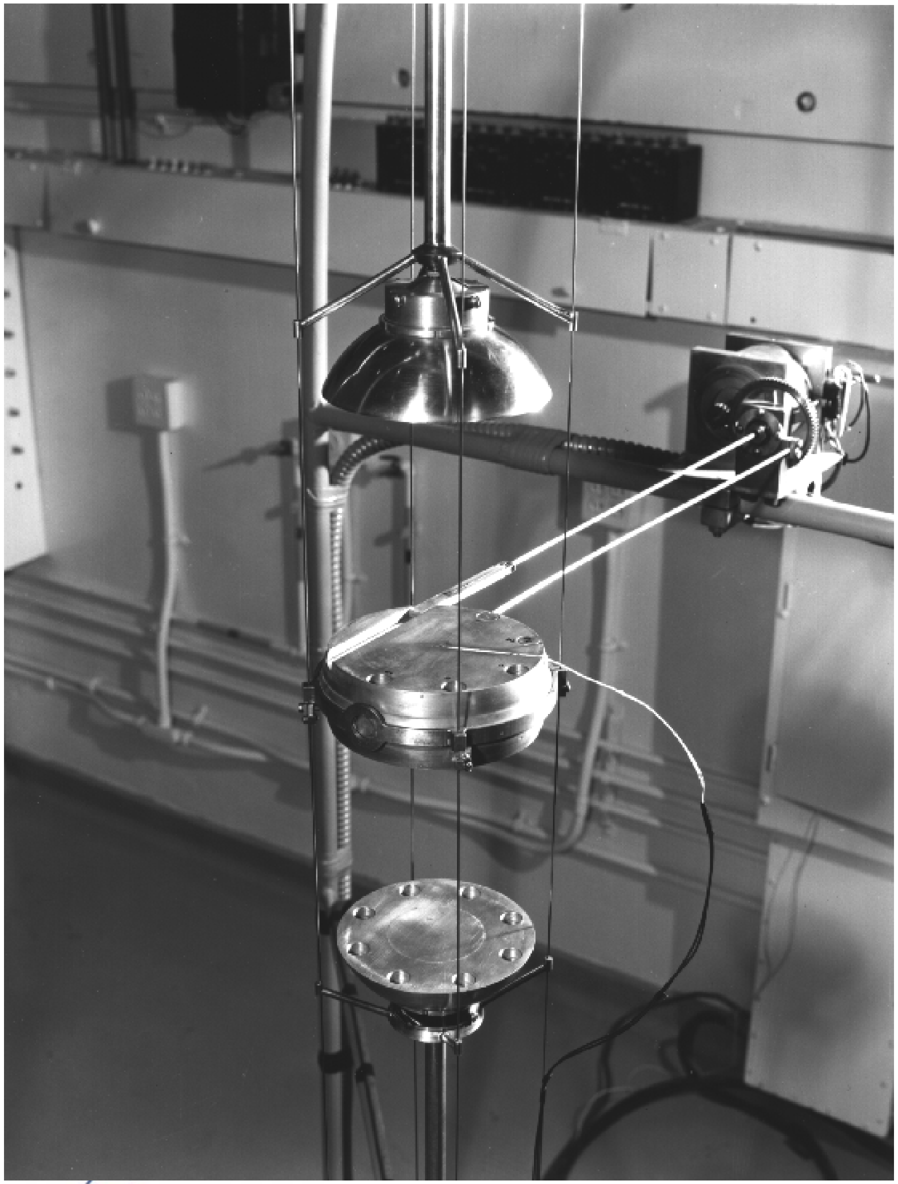
Nuclide	Atom Density (atoms/barn·cm)	Atom Fraction	Atom Fraction in Plutonium
Gallium	1.3752×10^{-3}	3.4132×10^{-2}	N/A
^{239}Pu	3.7047×10^{-2}	9.1951×10^{-1}	0.952
^{240}Pu	1.7512×10^{-3}	4.3465×10^{-2}	0.045
^{241}Pu	1.1674×10^{-4}	2.8975×10^{-3}	0.003

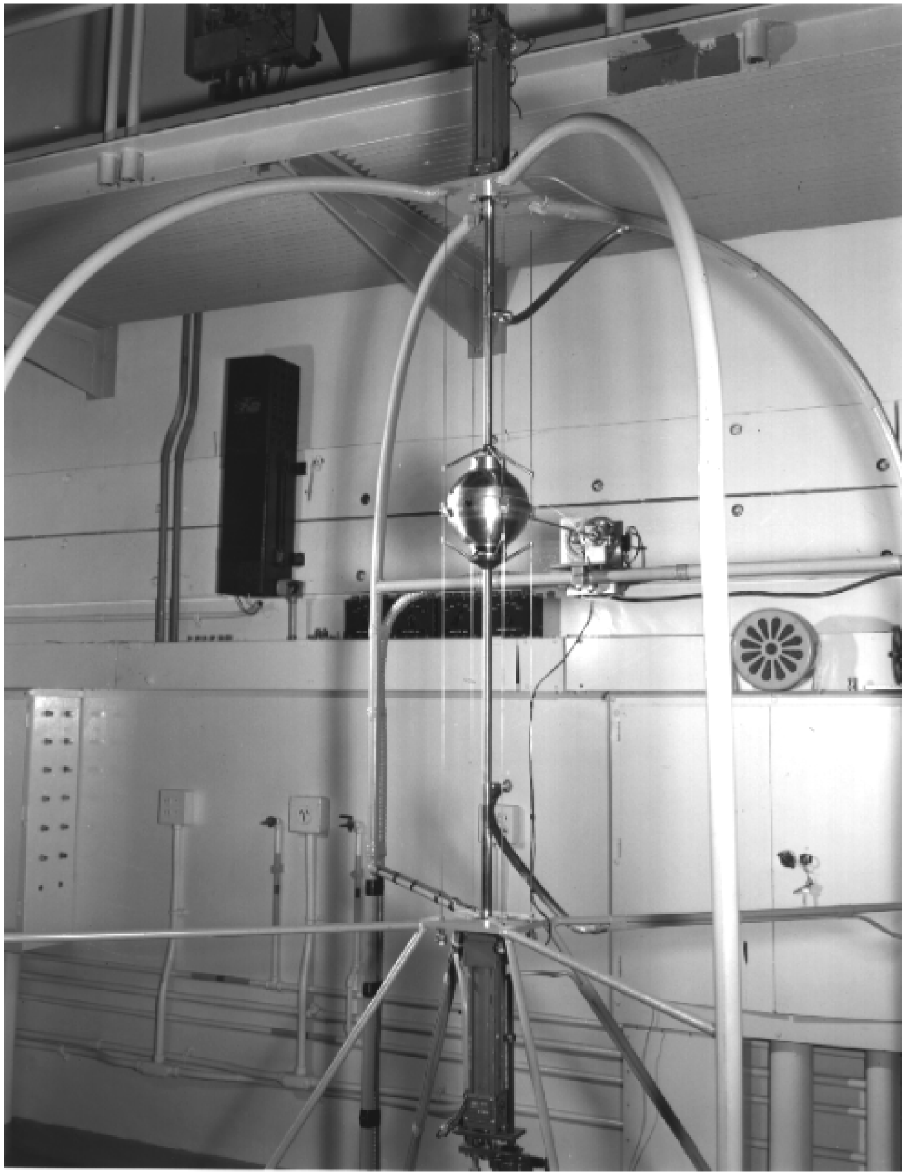
The actual Jezebel was more complicated…



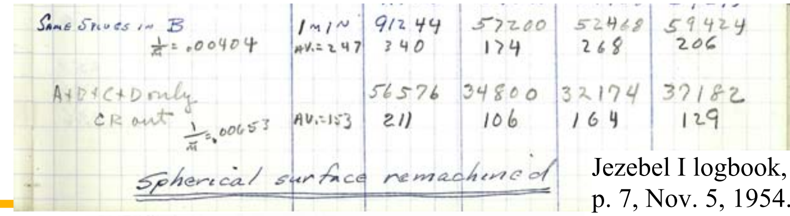
Drawing 19Y29288 C4, April 1952

Photos – Jan. 24, 1955

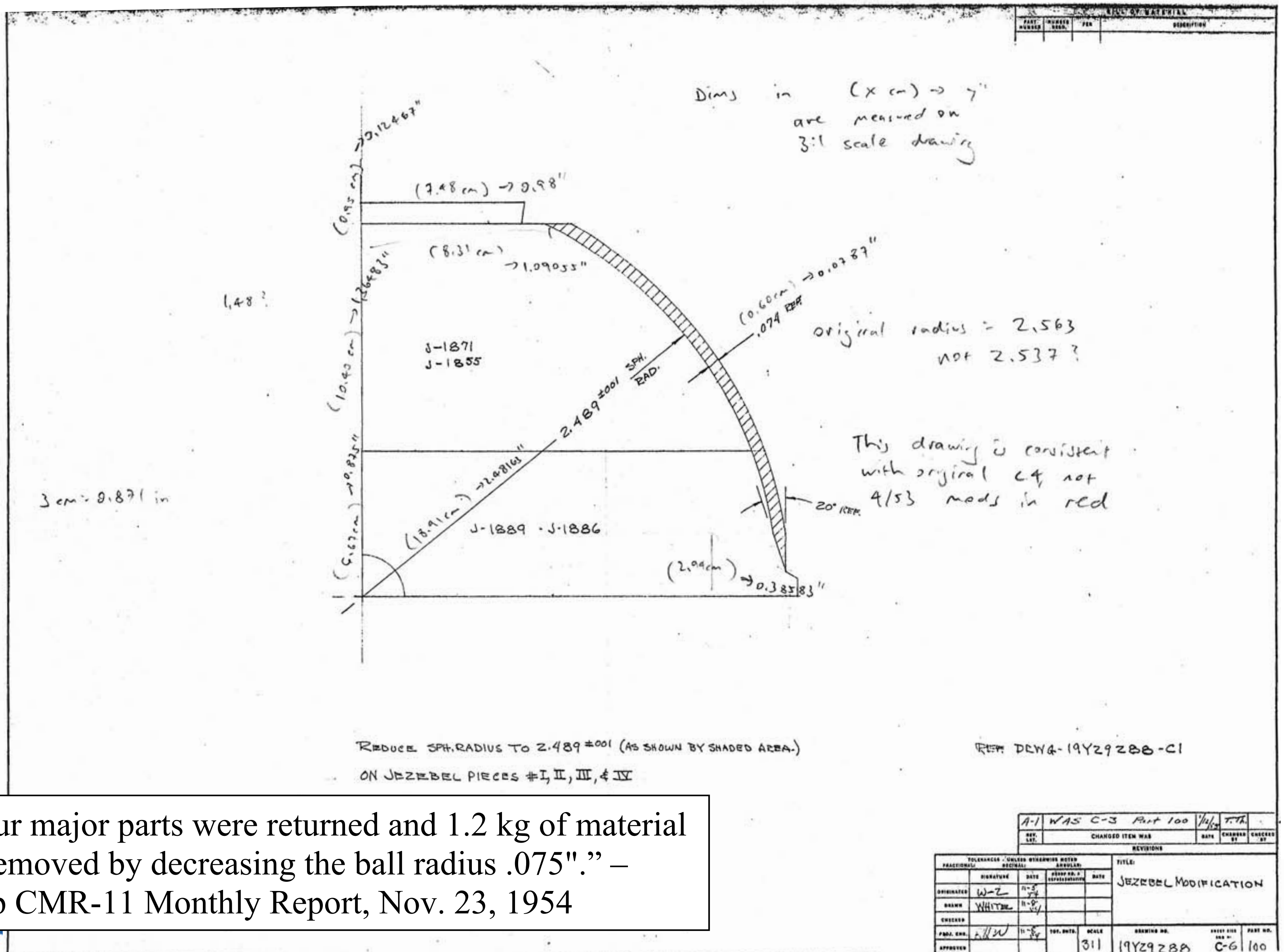




Major Parts Remachined in Nov. 1954



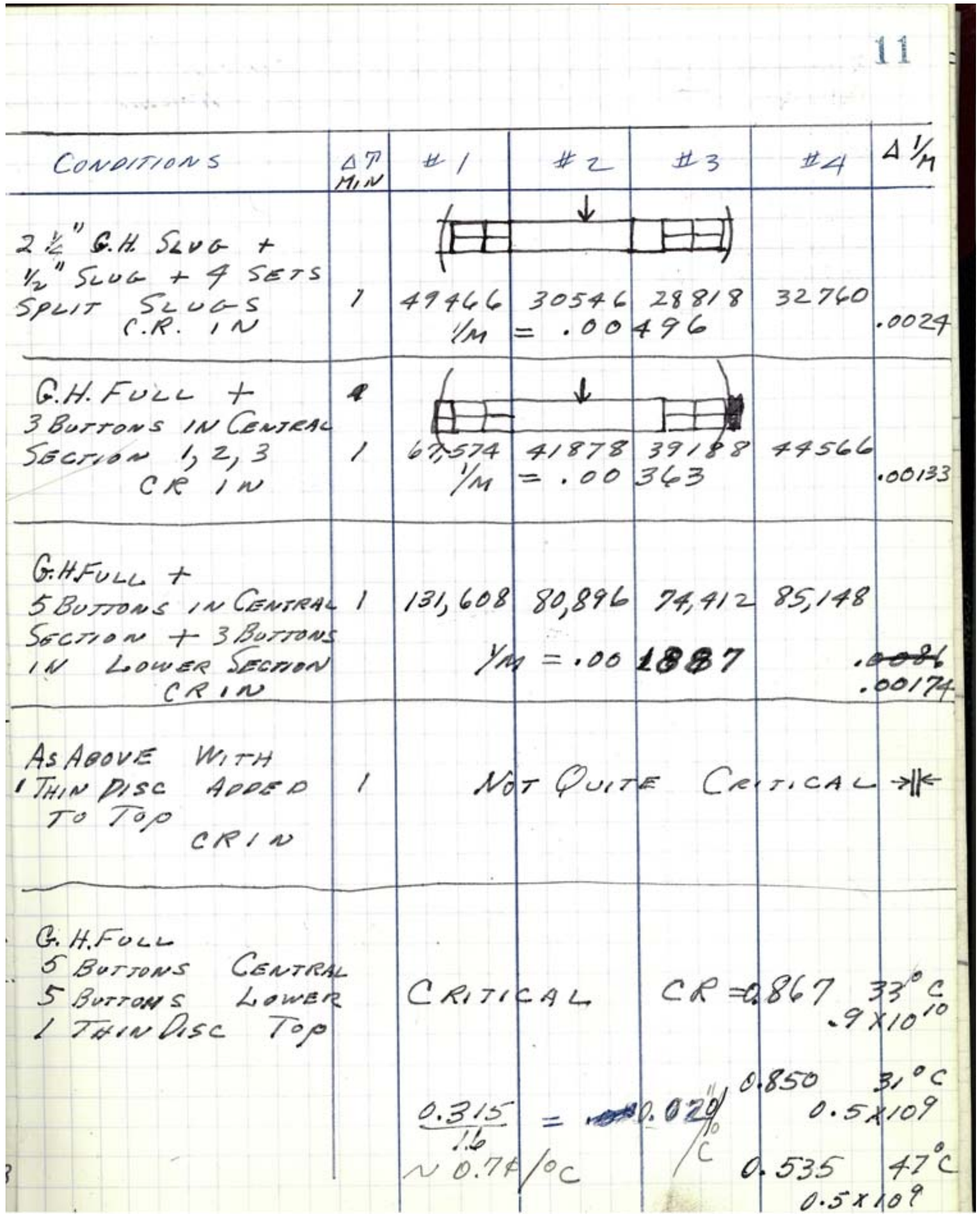
Jezebel I logbook, p. 7, Nov. 5, 1954.



Drawing 19Y29288 C6, Nov. 1954

"[F]our major parts were returned and 1.2 kg of material was removed by decreasing the ball radius .075"." – Group CMR-11 Monthly Report, Nov. 23, 1954

First Jezebel Criticality, Nov. 30, 1954



11

CONDITIONS	ΔT MIN	#1	#2	#3	#4	Δ 1/M
2 1/2" G.H. SLUG + 1/2" SLUG + 4 SETS SPLIT SLUGS C.R. IN	1	49466	30546	28818	32760	.0024
		1/M = .00496				
G.H. FULL + 3 BUTTONS IN CENTRAL SECTION 1, 2, 3 CR IN	1	67,574	41878	39188	44566	.00133
		1/M = .00363				
G.H. FULL + 5 BUTTONS IN CENTRAL SECTION + 3 BUTTONS IN LOWER SECTION CR IN	1	131,608	80,896	74,412	85,148	~~.0021~~ .00174
		1/M = .001887				
AS ABOVE WITH 1 THIN DISC ADDED TO TOP CR IN	1	NOT QUITE CRITICAL →‖←				
G.H. FULL 5 BUTTONS CENTRAL 5 BUTTONS LOWER 1 THIN DISC TOP		CRITICAL		CR = 0.867	33°C .9×10^10	
				0.850	31°C 0.5×10^9	
		0.315/16 = 0.020 ¢/°C ~ 0.74/°C		0.535	47°C 0.5×10^9	

Jezebel I logbook, p. 11, Nov. 30, 1954.

UNCLASSIFIED

Critical Mass: LA-2044 (1956) and *Nucl. Sci. Eng.* vol. 8 (1960)

- LA-2044[a] gave the critical mass of "Jezebel Pu alloy" (1 wt.% gallium) as 16.45 ± 0.05 kg at a density of 15.82 g/cm^3.

    + Like everything else, this report was classified; it was declassified in 1965.

- The *Nuclear Science and Engineering* paper[b] gave the critical mass of "a solid, bare sphere of Pu (4½ [at.]% Pu^{240})" as 16.28 ± 0.05 kg at a density of 15.66 g/cm^3.

    + The NSE paper failed to mention the 1 wt.% gallium….

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[a] G. A. Jarvis, G. A. Linenberger, and H. C. Paxton, "Plutonium-Metal Critical Assemblies," Los Alamos Scientific Laboratory report LA-2044, May 1956.
[b] G. A. Jarvis, G. A. Linenberger, J. D. Orndoff, and H. C. Paxton, "Two Plutonium-Metal Critical Assemblies," *Nucl. Sci. Eng.*, **8**, *6*, 525-531, December 1960.

LA-4208 (1969)[c]

- Described the development of a "reevaluated" one-dimensional model.

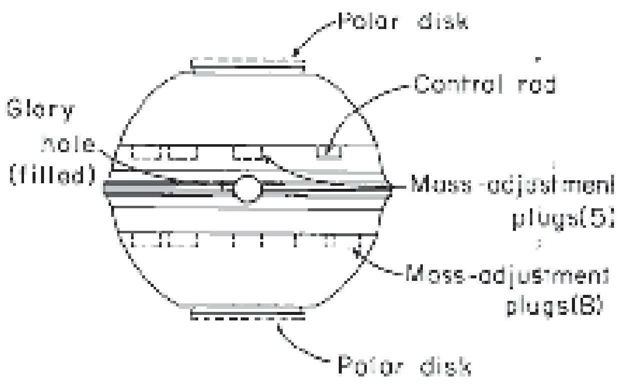


Fig. 6. Jezebel Pu (4.5% ^{240}Pu).
Configuration A, 16.751 kg alloy:

> *no polar disk; subcritical 0.43 lower mass-adjustment plug (or 10 g alloy at surface) with all mass-adjustment plugs in place and control rod fully inserted; critical mass is 16.761 kg alloy at average density 15.61 g/cm³.*

Configuration B, 16.909 kg alloy:

> *two polar disks; critical with 6 lower mass-adjustment plugs removed, and control rod retracted 1.375 in., with all mass-adjustment plugs in place and control rod fully inserted critical mass is 16.784 kg alloy at average density 15.60 g/cm³.*

Table I. JEZEBEL CORRECTIONS TO IDEALIZED SPHERES

	Pu(4.5% ^{240}Pu) Config. A	Pu(4.5% ^{240}Pu) Config. B
Critical mass, kg[a]	16.761	16.784
(Density, g/cm³)	(15.61)	(15.60)
Corrections, kg:		
Asphericity	-0.033	0.047
Internal Ni and homogenization	0.047[b]	0.033[c]
Equatorial band	0.045	0.045
Polar supports	0.117	0.117
External Ni	0.074	0.074
Framework	0.002	0.002
Kiva reflection	0.010	0.010
Air reflection	0.004	0.004
Trace impurities[e]	-0.001	-0.001
Elevated temp.	-0.007	-0.007
Critical mass of homogeneous sphere, kg alloy	17.019	17.014
(Density, g alloy/cm³)	(15.61)	(15.61)

17.02±0.6%
(15.61)

[a] Major cavities removed.

[b] Measured minus 144 g equivalent of 0.010-in.-thick Ni on one parting plane compares with calculated minus 142 g.

[c] Includes correction to $\rho = 15.61$ g/cm³.

[e] Pu impurities are about 600 ppm (170 ppm C, 230 ppm O, 115 ppm Fe); ^{233}U impurities are similar to those of Godiva.

- Converted to a k_{eff} uncertainty of ±0.002 (rounded up from ±0.00170) in PMF-001 Rev. 0.

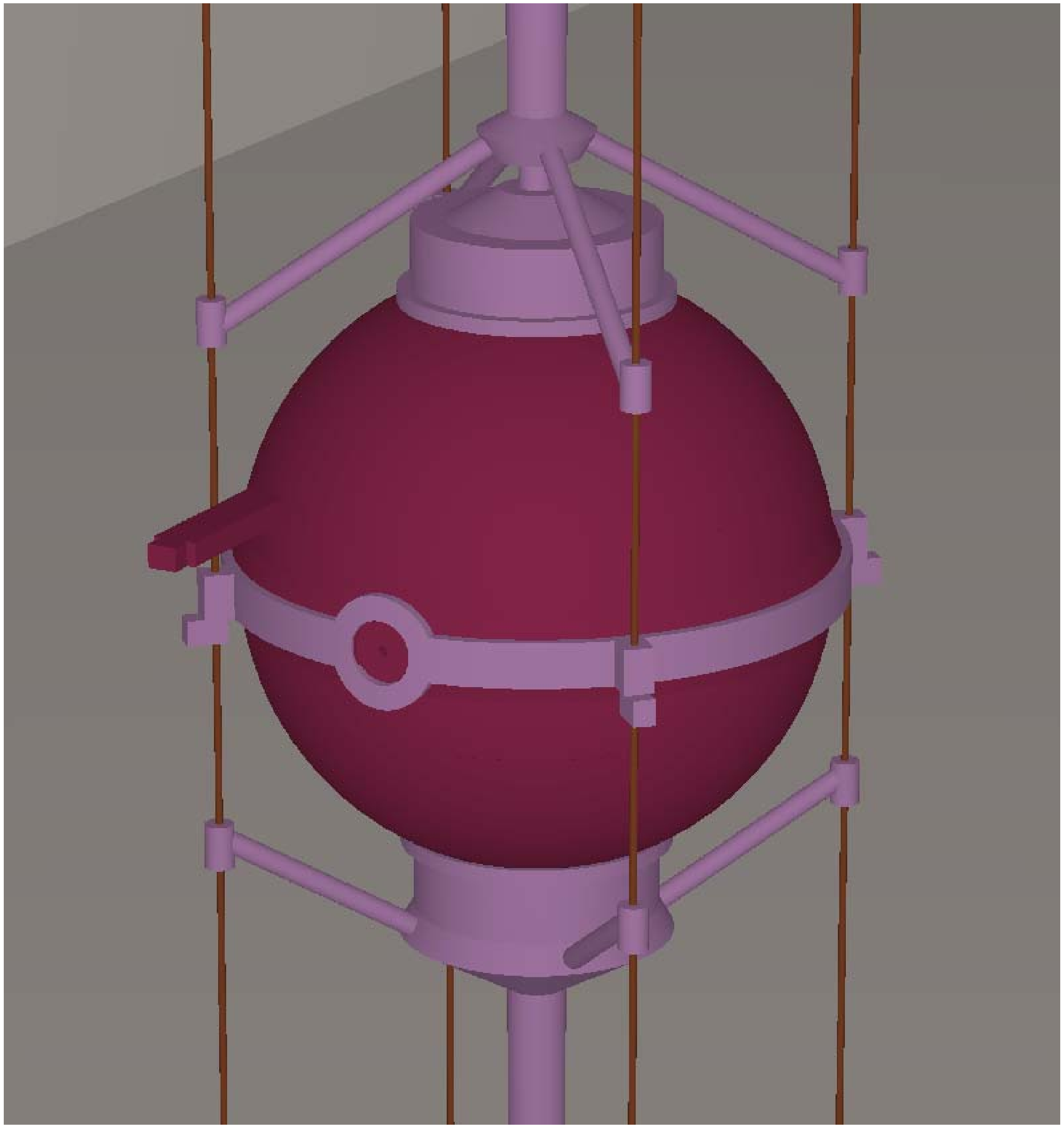
Jezebel Reevaluation for the ICSBEP Handbook (Rev. 3, 2013): Four Detailed Models

- Configurations A and B were described in LA-4208.[c] B was found in the logbook. C and D are from the logbook.

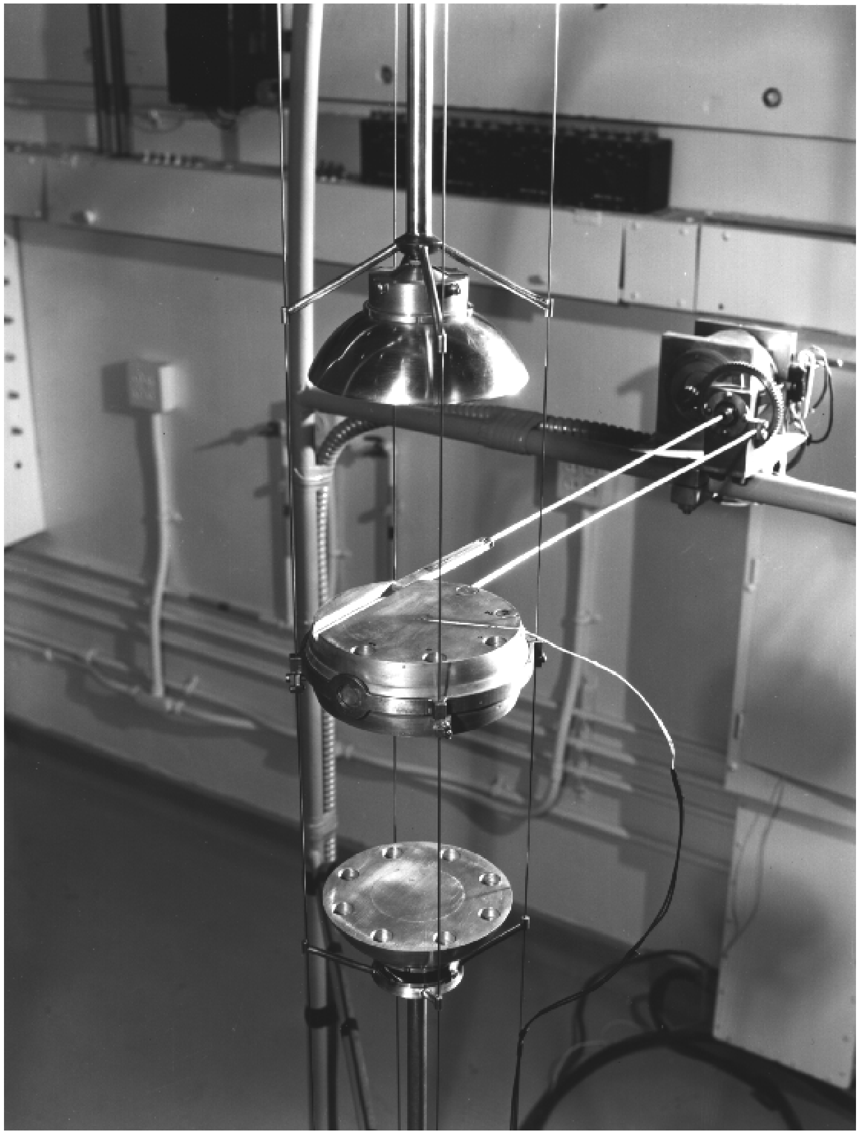
Configuration →	A	B	C	D
Experimental Assembly Mass (LA-4208) (kg Pu-alloy)	16.751	16.909	Not given	Not given
Model Assembly Mass (kg Pu-alloy)	16.752	16.908	16.829	16.865
Average Pu-alloy Density (g/cm^3)	15.78	15.78	15.78	15.78
Control Rod Position	Fully inserted	Retracted 1.375 inches	Retracted 0.867 inch	Retracted 1.276 inches
Mass Adjustment Buttons in Upper Part M3	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5
Mass Adjustment Buttons in Lower Part M2	6, 7, 8, 9, 10, 11, 12, 13	6, 7	6, 8, 10, 11, 13	6, 7, 8, 9, 10, 11, 12, 13
Glory Hole	Full	Full	Full	Full
Thin Polar End Caps (Upper and/or Lower M1′)	None	Upper and lower	Upper	Lower
Al Spacer Ring	Present	Present	Present	Present
Thick Polar End Caps (M1)	None	None	None	None

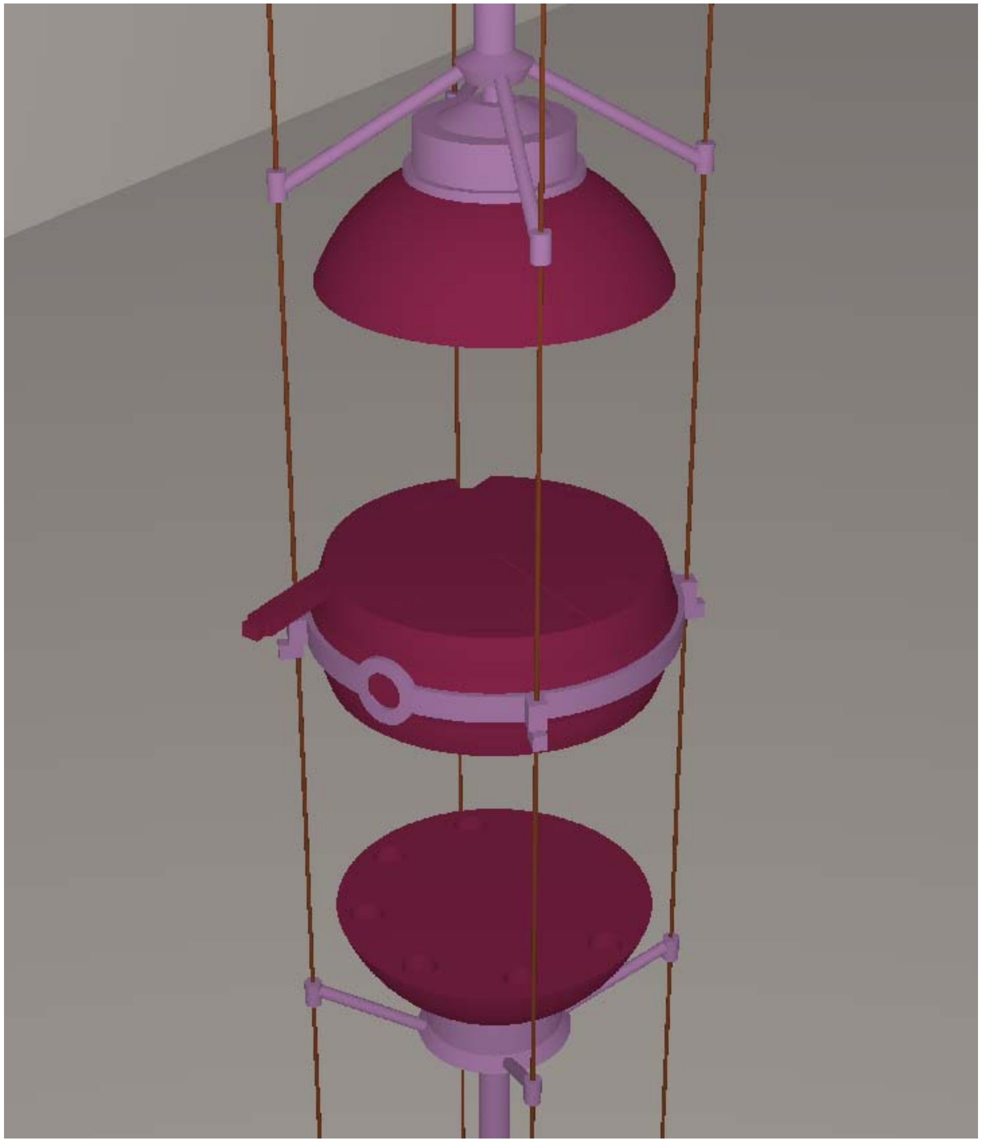
[c] G. E. Hansen and H. C. Paxton, "Reevaluated Critical Specifications of Some Los Alamos Fast-Neutron Systems," Los Alamos Scientific Laboratory report LA-4208, September 1969.

MCNP Visual Editor Rendering of Configuration B (1 of 2)

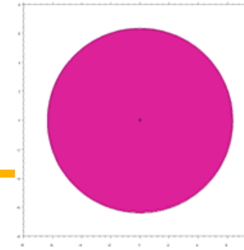


MCNP Visual Editor Rendering of Configuration B (2 of 2)



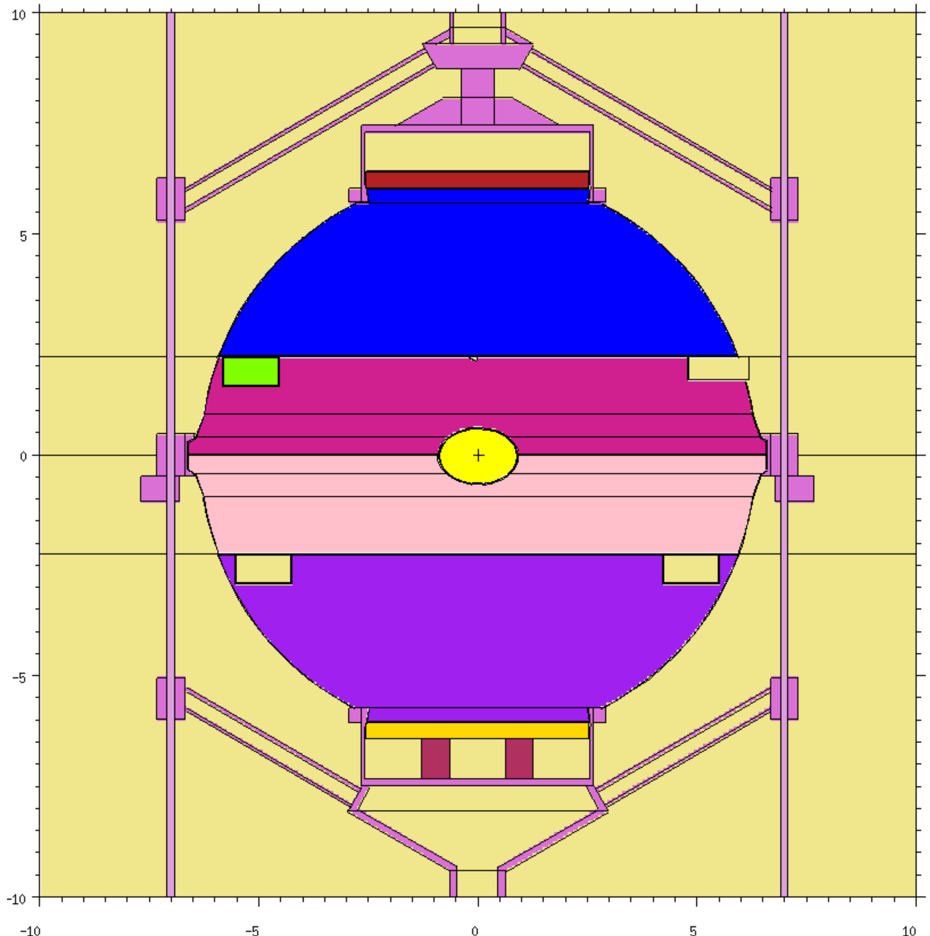


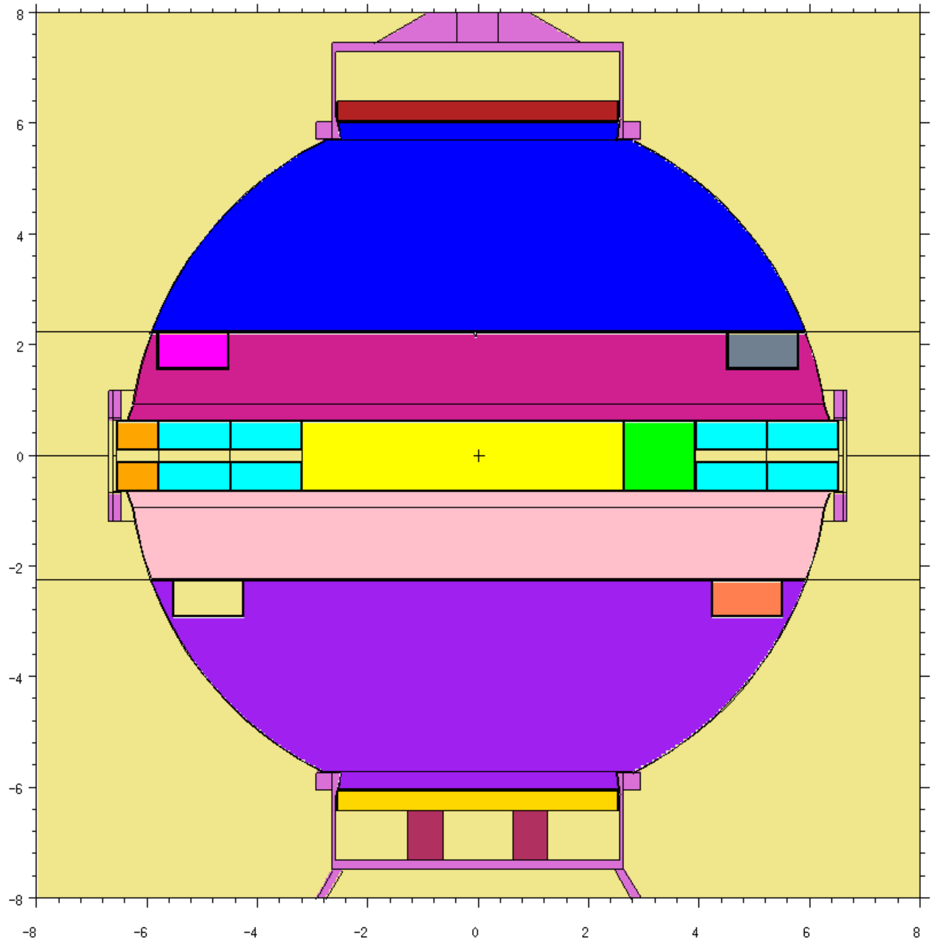
MCNP Renderings of Configuration B



Spider assemblies, piano wire, belly band, wire lugs and clamps, control rod, mass adjustment buttons

Glory hole fill, mass adjustment buttons, external and internal nickel, thin polar end caps, aluminum shim





Detailed Model k_{eff}'s and a New One-Dimensional Model: PU-MET-FAST-001, Rev. 3

	Experimental k_{eff}	Calculated k_{eff}	Calc./Exp.
Config. A	0.99999 ± 0.00129	1.00072 ± 0.00002	1.00073 ± 0.00129
Config. B	1.00016 ± 0.00129	1.00115 ± 0.00002	1.00099 ± 0.00129
Config. C	1.00020 ± 0.00129	1.00094 ± 0.00002	1.00074 ± 0.00129
Config. D	1.00128 ± 0.00129	1.00190 ± 0.00002	1.00062 ± 0.00129
Average	–	–	1.00077 ± 0.00016

- The benchmark one-dimensional model was redefined to be the one that gives k_{eff} = 1.00077 when ENDF-B/VII.1 cross sections are used.
  - Mass = 17,073.2 ± 77 g Pu-alloy
  - Density = 15.61 g/cm^3, same as previous benchmark (and the material is the same)
  - Benchmark k_{eff} = 1.00000 ± 0.00129

- The reevaluated one-dimensional benchmark, 17.0732 kg ± 0.077 kg Pu-alloy, is statistically indistinguishable from the previous one-dimensional benchmark, 17.02 kg ± 0.6%.

Assumption 1: Assembly Masses in LA-4208 Are Correct

- LA-4208 gave assembly masses for Configurations A and B.

- The earliest mass accountability statements (giving masses for individual parts) that have been found are from 1960.

- The logbooks describe an episode of nickel replating in Nov. 1958 in which one of the major parts lost 72.69 g.

- Adding the 1960 masses for Configurations A and B, and adding the mass lost in the nickel replating of Nov. 1958, the totals are ~169 g less than the LA-4208 masses.
  - \+ We have assumed some other undocumented process (perhaps nickel replating) in which the other three major parts lost a total of ~163 g.

- The control rod (plutonium) was replated in Nov. 1958 and "recoated" in Nov. 1957 but its new mass was not recorded either time.
  - \+ Using the 1960 mass statements, the control rod density is 14.34 g/cm^3.
  - \+ We added 5.58 g to the control rod to bring its density to 15.61 g/cm^3.

- We distributed the remaining ~163 g equally among the three major parts that were not replated in Nov. 1958.

- What is the uncertainty associated with the uncertain mass distribution?

Mass Accountability Statement, Aug. 31, 1960

"N-255 Materials Inventory"
"8/31/60" handwritten top
p 8 of 22

PLUTONIUM

JEZEBEL

J-1970	LOWER SAFETY BLOCK 2.563	3926.74	Lower M2
J-1855-A	UPPER SAFETY BLOCK 2.563	4001.50	Upper M2
J-188361858	LOWER CENTER SECTION 2.563	4159.29	Lower M3
J-188961886	UPPER CENTER SECTION 2.563	3975.23	Upper M3
J-1886-A2	MASS DISC 2 D. X 5616 T	264.83	
J-1890-A	MASS DISC 2 D. X 5616 T	250.31	
J-1890-B	MASS DISC 2 D. X .141 T	112.14	
J-1893-A1	MASS ADJ. PLUG .485 X .243	11.43	
J-1893-A2	MASS ADJ. PLUG .485 X .243	11.52	
J-1893-A3	MASS ADJ. PLUG .485 X .243	11.25	
J-1893-A4	MASS ADJ. PLUG .485 X .243	11.58	
J-1893-A5	MASS ADJ. PLUG .485 X .243	11.54	
J-1893-A6	MASS ADJ. PLUG .485 X .243	11.61	
J-1893-A7	MASS ADJ. PLUG .485 X .243	11.56	
J-1893-A8	MASS ADJ. PLUG .485 X .243	11.65	
J-1893-A9	MASS ADJ. PLUG .485 X .243	11.57	
J-1893-A10	MASS ADJ. PLUG .485 X .243	11.62	
J-1893-A11	MASS ADJ. PLUG .485 X .243	11.68	
J-1893-A12	MASS ADJ. PLUG .485 X .243	11.52	
J-1893-A13	MASS ADJ. PLUG .485 X .243	11.49	
JX-1893-A14	GH SLUG .4885 X .498	23.76	
JX-1903-A1	GLORY HOLE SPLIT SLUG	11.27	
JX-1903-A2	GLORY HOLE SPLIT SLUG	10.97	
JX-1903-A3	GLORY HOLE SPLIT SLUG	11.33	
JX-1903-A4	GLORY HOLE SPLIT SLUG	11.01	
JX-1903-A5	GLORY HOLE SPLIT SLUG	11.17	
JX-1903-A6	GLORY HOLE SPLIT SLUG	11.07	
JX-1903-A7	GLORY HOLE SPLIT SLUG	11.19	
JX-1903-A8	GLORY HOLE SPLIT SLUG	11.11	
JX-1903-A9	GLORY HOLE SPLIT SLUG	11.14	
JX-1903-A10	GLORY HOLE SPLIT SLUG	11.01	
JX-1903-A11	GLORY HOLE SPLIT SLUG	6.24	
JX-1903-A12	GLORY HOLE SPLIT SLUG	6.29	
JX-1903-A13	CONTROL ROD 3.715 LONG	63.15	Control rod
JX-1903-B	CONTROL ROD 2.287 LONG	108.41	
J-1905-A1	MASS DISC 2 D. X .141 T	113.31	
JX-1972-A1	DISC COATED	10.17	
JX-1972-A2	DISC COATED	10.18	
JX-1972-A3	DISC COATED	10.10	
JX-1972-A4	DISC COATED	10.05	
JX-1972-A5	DISC COATED	10.07	
JX-1972-A6	DISC COATED	10.24	
JX-1972-A7	DISC COATED	9.97	
JX-1972-A8	DISC COATED	10.03	
JX-1972-A9	DISC COATED	10.01	
JX-1972-A10	DISC COATED	10.13	
[illegible]	MASS ADJ. PART	47.02	

Plutonium Mass, Dimensions, and Density Uncertainties

- Linear dimensions were taken from drawings.

- Densities were not given for the individual parts (the average density was 15.82 g/cm^3).
  + LA-4208: the density of the "major parts [was] measured with a precision of ±0.2%."
  + During this period, mass could have been measured to less than a milligram. For many parts, mass is given to the nearest 0.01 gram.
  + Thus, the volume was measured to 0.2%.

- The relative uncertainty in k_{eff} due to correlated mass and volume uncertainties for each part independently is[d]

$$\left(\frac{\delta k_{eff}}{k_{eff}}\right)^2 = S_{k,\rho_d}^2\left[\left(\frac{u_{m_d}}{m_d}\right)^2 + \left(\frac{u_{V_d}}{V_d}\right)^2\right] + \left(\frac{V_d}{k_{eff}}\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}^2\left(\frac{u_{V_d}}{V_d}\right)^2 - 2S_{k,\rho_d}\left(\frac{V_d}{k_{eff}}\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d}\left(\frac{u_{V_d}}{V_d}\right)^2,$$

with $S_{k,\rho_d} \equiv \frac{\rho_d}{k_{eff}}\left(\frac{\partial k_{eff}}{\partial \rho_d}\right)_{V_d}$ and $\left(\frac{\partial k_{eff}}{\partial V_d}\right)_{\rho_d} = \frac{\sum_{n=1}^{N_d}\left(\partial k_{eff}/\partial r_n\right)_{\rho_d;\, r_m, m\neq n}}{\sum_{n=1}^{N_d}\left(\partial V_d/\partial r_n\right)_{r_m, m\neq n}}$,

where N_d is the number of linear dimensions describing part *d*.

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[d] J. A. Favorite, J. C. Armstrong, and T. Burr, "Uncertainty Analysis of Densities and Isotopics: Handling Correlations," *Proceedings of the International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering (M&C 2013)*, CD-ROM, Sun Valley, Idaho, May 5-9, 2013.

Plutonium Mass Distribution Correlations (Total Mass σ = ±2 g)

- The three large parts and the control rod, among which the "missing" 169 g was distributed, are correlated. The total $\delta k_{eff}/k_{eff}$ for the four parts is

$$\left(\frac{\delta k_{eff}}{k_{eff}}\right)^2 = \sum_{i=1}^{4} S_{k,\rho_i}^2 \left(\frac{u_{m_i}}{m_i}\right)^2 + 2\sum_{i=1}^{3}\sum_{j=i+1}^{4} S_{k,\rho_i} S_{k,\rho_j} \left(\frac{u_{m_i}}{m_i}\right)\left(\frac{u_{m_j}}{m_j}\right) r_{i,j},$$

where $r_{i,j}$ is the usual correlation coefficient, $r_{i,j} \equiv \mathrm{cov}(m_i, m_j)/\left(u_{m_i} u_{m_j}\right)$, and the covariance for M independent observations of m_i and m_j is $\mathrm{cov}(m_i, m_j) = \frac{1}{M-1}\sum_{l=1}^{M}(m_{i,l} - \overline{m}_i)(m_{j,l} - \overline{m}_j)$, where $\overline{m}_d$ is the average mass of part d for the M observations.

- $M = 1 \times 10^6$ mass distributions were randomly generated.
  - \+ A mass to distribute was sampled from a Gaussian (169 ± 2 g);
  - \+ From 0 to 11.16 g was added to the control rod (random, uniform);
  - \+ The rest was distributed (randomly, uniformly) among the "big 3";
  - \+ Densities were not allowed to be less than 15.15 or greater than 16.41 g/cm^3.

Part	Base mass (g)	Mean (g)	Std. Dev. (g)	Std. Dev./Mean
Upper M2	4055.88	4055.5953	29.2222	0.7205%
Lower M2	3981.12	3980.8878	29.1966	0.7334%
Lower M3	4213.67	4213.4332	29.2049	0.6931%
Control rod	68.73	69.4841	1.6001	2.3028%

k_{eff} Uncertainty Due to Pu Mass, Dims., and Densities (4 Parts)

- Results from 200 k_{eff} calculations for three cases of total mass uncertainty:

Total mass σ	Conf.	Base k_{eff}	Mean	Std. Dev	Difference Between Mean and Base k_{eff}
±0 g	A	1.00072	1.00071	0.00049	–0.00001
	B	1.00115	1.00114	0.00046	–0.00001
±2 g	A	1.00072	1.00080	0.00052	0.00008
	B	1.00115	1.00122	0.00049	0.00007
±10 g	A	1.00072	1.00070	0.00065	–0.00002
	B	1.00115	1.00113	0.00064	–0.00002

- The brute-force calculations did not include the volume uncertainty of 0.2%.

- Using $u_{V_d}/V_d = 0\%$ in the equation for $\delta k_{eff}/k_{eff}$ for Conf. B, and using only the four parts,
  - ±0 g → $\delta k_{eff}/k_{eff} = \pm 0.00047$
  - ±2 g → $\delta k_{eff}/k_{eff} = \pm 0.00047$
  - ±10 g → $\delta k_{eff}/k_{eff} = \pm 0.00067$

- CONCLUSION:
  - The uncertainty in the mass to distribute does not add much to the total uncertainty;
  - Or, the distribution of the mass is far more important than how much there is to distribute.

k_{eff} Uncertainty Due to Pu Mass, Dims., and Densities (All Parts; Rev. 3)

- Results from equations:

Part		$\delta k_{eff}/k_{eff}$		
	Total mass σ ±0 g	Total mass σ ±2 g	Total mass σ ±10 g	No unc. due to mass distribution
Upper M2	±0.00127	±0.00127	±0.00128	±0.00021
Lower M2	±0.00128	±0.00128	±0.00129	±0.00021
Upper M3			±0.00035	
Lower M3	±0.00173	±0.00173	±0.00174	±0.00034
Upper M1′			±0.00000	
Lower M1′			±0.00000	
Control rod[a]	±0.00005	±0.00005	±0.00005	±0.00000
GH filler[a]			±0.00003	
Buttons[a]			±0.00000	
Cross terms	-5.81×10^{-6}	-5.82×10^{-6}	-5.68×10^{-6}	0.00
Total mass	0.00	+0.00002	+0.00003	0.00
Total	±0.00074	±0.00076	±0.00091	±0.00057

[a] Density uncertainty only.

Material Transfer Receipt, Nov. 24, 1954

FORM NO. [illegible]

SF MATERIAL TRANSFER RECEIPT

Use separate receipt for each material type.

White copy to Group ADP-SF Office at time of transfer.
Yellow copy to person receiving material.
Pink copy to person from whom material is received.

Unit of Measurement g (Grams, Pounds, etc.) Type of Material Pu (U-235, U-238, Pu, etc.)

IDENTIFICATION NUMBER	COMPOUND, FORM, OR OTHER DESCRIPTION	NET WT. OF MATERIAL	SF NET OR [illegible] WEIGHT	PERCENT ENRICHMENT	U-235 WEIGHT
Jezebel #1	✓ W/o W H	3966 40	3986 74		
#3 ~~#2~~	✓	4088 57	4047 92	— Reworked	5/1968
#2 ~~#3~~	✓	4201 30	4159 29	—	"
#4	✓	4091 74	4061 50		
J-1903H10	✓	11 49	11 01		split slug
J-1903H11	✓	6 47	6 41		} "split slugs"
J-1903H12	✓	6 25	6 39		
J-1903H13	✓	63 91	63 27	out 1/18/57. [illegible]	
J-1593H9	✓	11 49	11 57		
J-1593H10	✓	11 24	11 13		
J-1593H11	✓	11 50	11 [illegible]		SMP
J-1593H12	✓	11 84	11 59		
J-1593H13	✓	11 61	11 49		
J-1593H14	✓	[illegible]	22 16		½" E. P. piece
J-1593H-1	✓	11 35	11 21		split slug
Totals			16315 84		

Transferred to: W-2 R. White (Group) Acct. No. 402 Date 11/24/54

Transferred from: (Group) Acct. No. 214 [illegible] Issued by

Transferred by JCW 11-24-54 R. White Received by

Lower M2

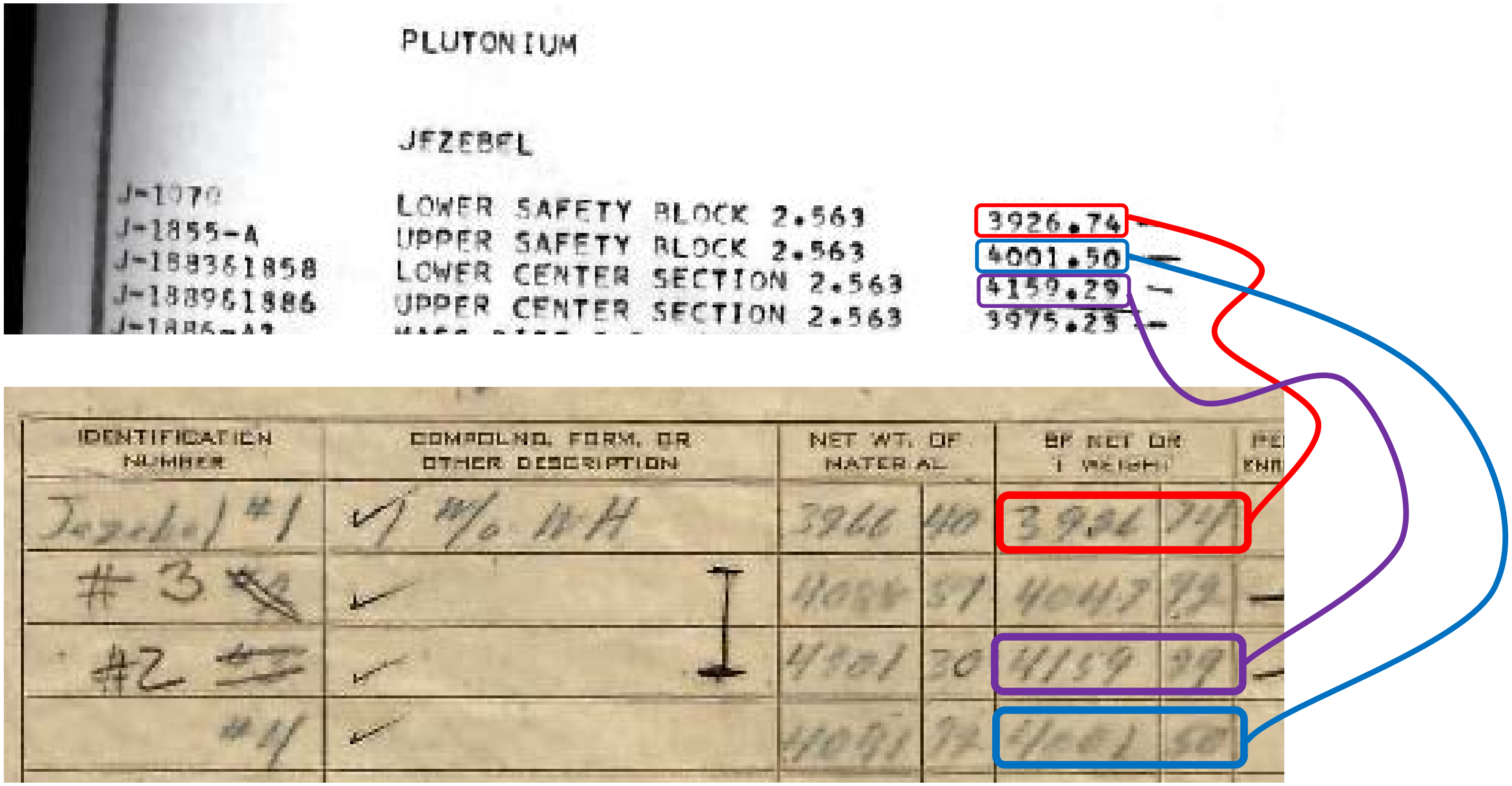
Upper M3

Lower M3

Upper M2

Control rod

Part Masses Compared



Conclusion: The mass accountability statement lists plutonium mass, not part mass!

Changes in the Benchmark Part Masses

Part	Rev. 3 Benchmark Mass (g)	New Mass (g)	Difference Relative to Rev. 3
Upper M3	4047.92	4088.81	1.010%
Lower M3	4213.67	4201.30	–0.294%
Upper M2	4055.88	4041.92	–0.344%
Lower M2	3981.12	3966.40	–0.370%
Upper Part M1′	112.14	113.30	1.034%
Lower Part M1′	113.31	114.47	1.024%
Control rod	68.73	63.91	–7.013%
2.287" GH filler piece	108.41	109.53	1.033%
0.5" solid GH filler piece	23.76	24.00	1.010%
0.5" split GH filler piece (pair)	22.33	22.56	1.030%
0.287" split GH filler piece (pair)	12.70	12.82	0.945%
Average button	11.58	11.70	1.036%

- Using the new masses in the Rev. 3 dimensions (average C/E is 0.00055 greater than Rev. 3 C/E):

Conf.	Calc./Exp.
A	1.00125 ± ~~0.00116~~
B	1.00156 ± ~~0.00116~~
C	1.00129 ± ~~0.00116~~
D	1.00118 ± ~~0.00116~~
Avg.	1.00132 ± 0.00017

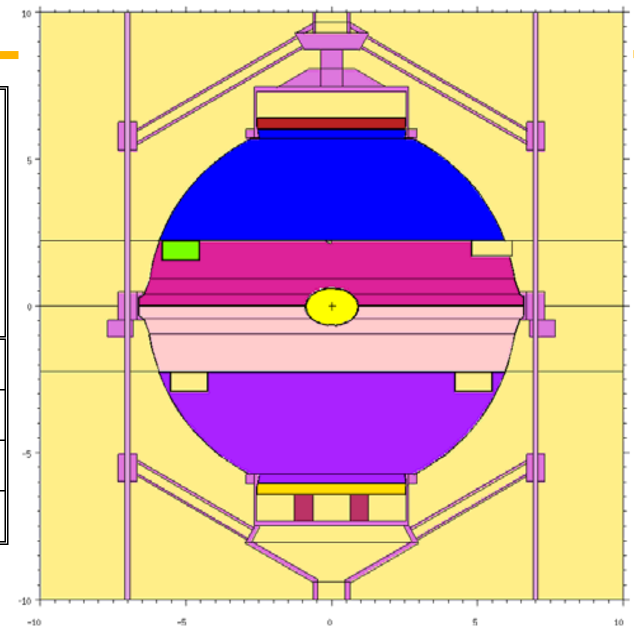
Unofficial Handwritten Notes (with Density!)

JEZEBEL

	Part #'s & Description		wts 11/24/54	11/6/58	5/1/68	wts 4/21/69		Density
Lower M2	#1 J-1070 + J- ? Lower Safety Block	Coated Alloy "T"	– 3966.40 3926.74			3966.40 3926.74		2 kilogram casting J-1070 12/18/50 was 15.751
Upper M3	#2 J-1889 + J-1886 Upper Center	Coated Alloy "T"	– 4088.81 4049.92	4057.61 4015.38 3975.23	– 3995.00 3955.00	– 3995.00 3955.00		?
Lower M3	#3 J-1883 + J-1858A Lower Center	Coated Alloy "T"	4239.50 4201.30 4159.29		4180.00 4138.00	4180.00 4138.00		15.830
Upper M2	#4 J-1855A + J-1871A Upper Safety	Coated Alloy "T"	4041.92 4001.50			4042.10 4002.00		15.782
		Total "T" wt.	16137.45			16021.24	Diff	116.21 Grams

9 should be 7

What to Do with these Densities?



Part	Rev. 3 Benchmark Density (Calculated; g/cm^3)	Density Using Updated Masses (Calculated; g/cm^3)	Density from Notes (Measured?; g/cm^3)	Volume Change Needed
Upper M3	15.9045	16.0652	"?"	1.485%[b]
Lower M3	15.9797	15.9328	15.830	0.650%
Upper M2	15.5753	15.5217	15.782	–1.650%
Lower M2	15.7082	15.6501	15.751[a]	–0.640%

[a] Value for one of the 2-kg castings, not the whole part.
[b] Assuming the density is 15.830 g/cm^3.

- Good: Densities in the notes are more consistent with each other and with handbook value.
- Bad: There is no record of the volume measurements that must have been done; the reference for these notes is "Author unknown, handwritten notes, sometime after 4/20/1969."
- Bad: The volume changes require dimension changes well outside the tolerances on the drawings, by factors of ~5-12.
    + The volume uncertainty used in the benchmark was that claimed in LA-4208 for densities, 0.2%.
- What densities to use? What dimensions to use?

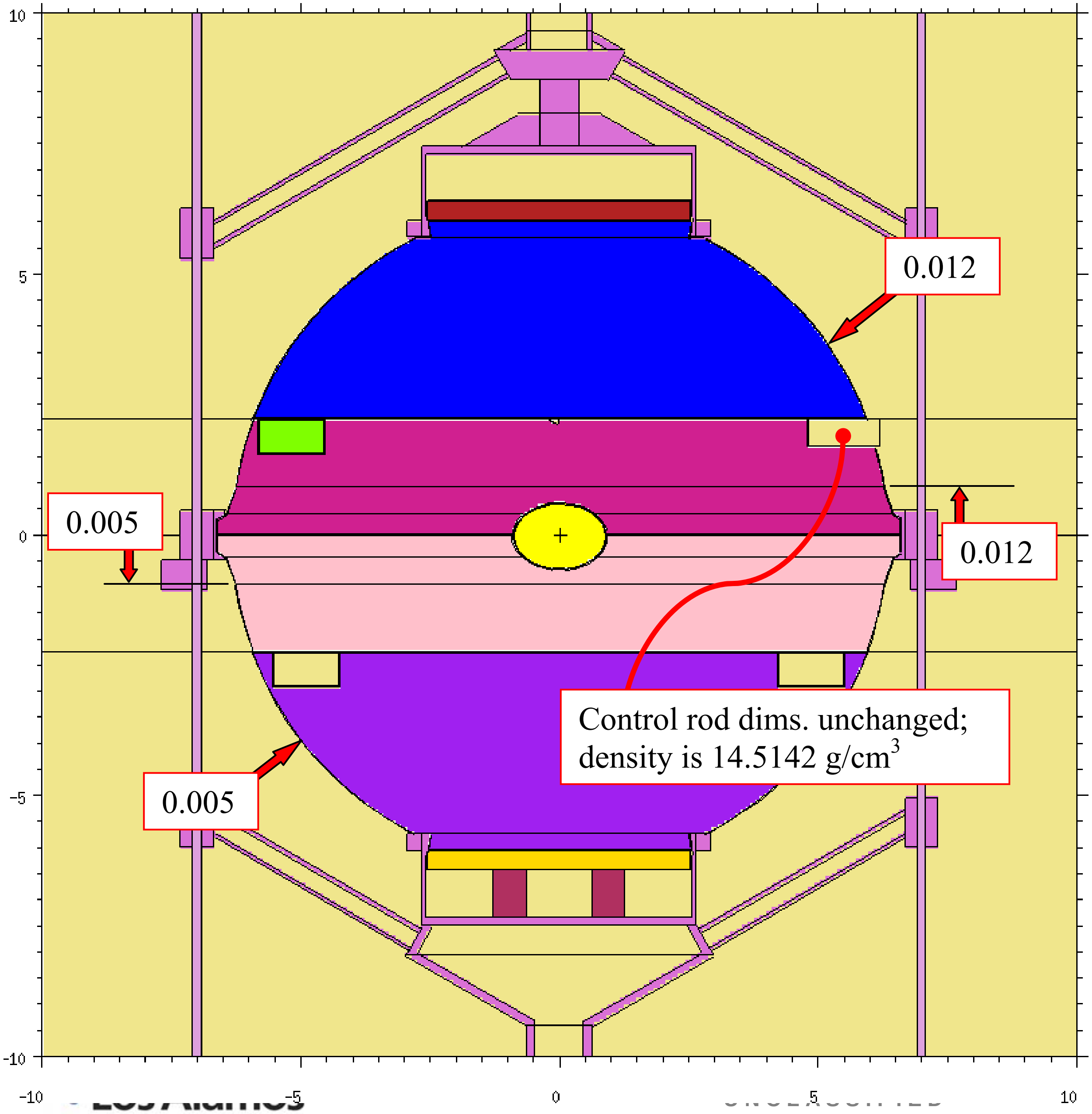
The Control Rod is a Particular Pain

- In Rev. 3, we added 5.58 g to the control rod because otherwise its density would be too low (using the mass from Aug. 31, 1960 of 63.15 g and the drawing dimensions).
  + We assumed that mass was lost in nickel replating episodes between 1954 and 1960.

- We now know the 1954 control rod mass was 63.91 g. We do not know the dimensions.

Source of dimensions	Control rod length (inch)	Density (Calculated; g/cm^3)
Original drawing	3.715 ± 0.005	14.5142
Modified drawing ("red")	3.645 ± 0.002	14.7843
Reduce length in original drawing by 7.24%	3.4460	15.61
Reduce length in original drawing by 8.27%	3.4076	15.78

Dimensions Changed (inches): Round 1

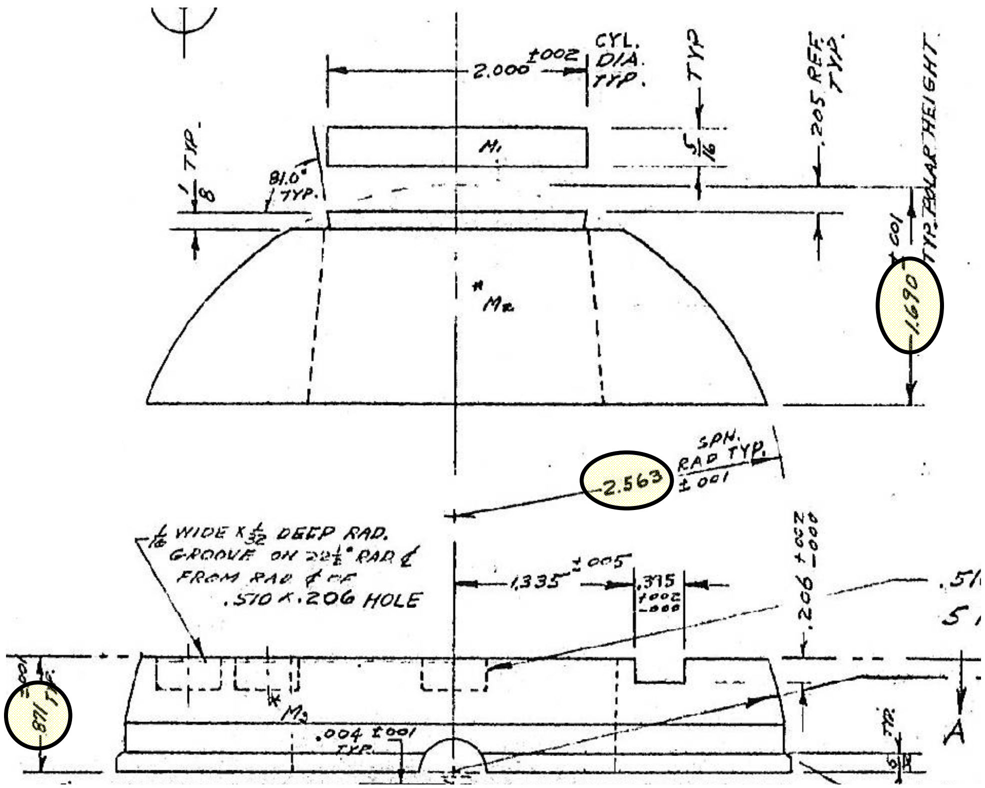


- Average C/E is 0.00066 less than Rev. 3 C/E:

Conf.	Calc./Exp.
A	1.00002 ± ~~0.00129~~
B	1.00038 ± ~~0.00129~~
C	1.00007 ± ~~0.00129~~
D	0.99997 ± ~~0.00129~~
Avg.	1.00011 ± 0.00018

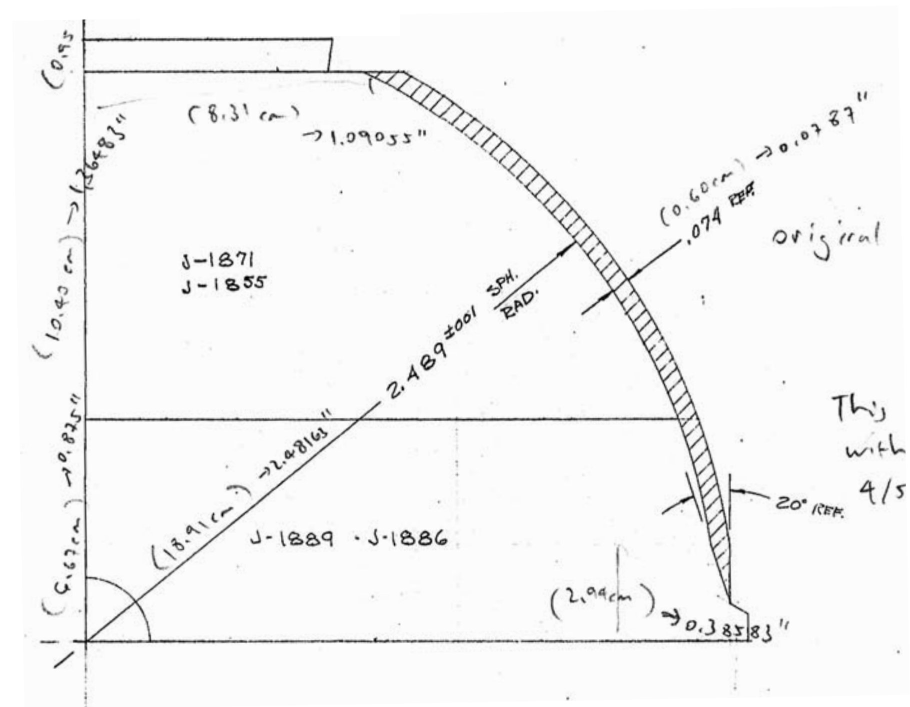
Ambiguity in the Dimensions (M2 Parts)

19Y29288 C4, April 1952



NOTE: ALL DIM. GIVEN APPLY TO UNPLATED COMPONENTS. .004 PLATE MAX. CONTEMPLATED.

19Y29288 C6, Nov. 1954



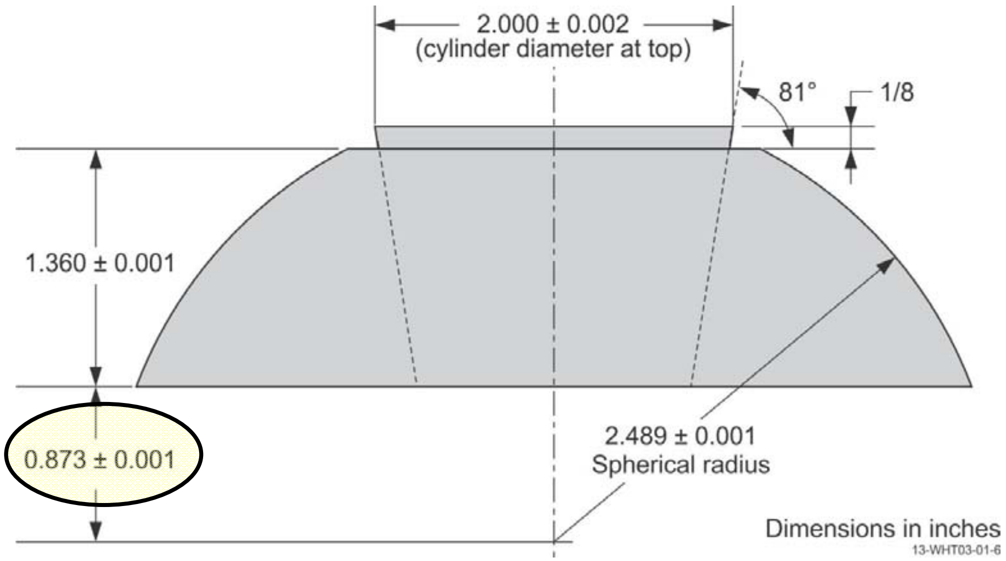
No note about unplated vs. plated components – assumed unplated.

Assumption 2: Center of M2 Parts is 0.873 in. from the Parts

- The original drawing implies the distance from the M2 spherical center to the bottom surface is 0.873 inch.

- The drawing (Nov. 1954) showing the remachining of Nov. 1954 implies the distance is the same as the height of the M3 parts, 0.871 inch.

- **Assumed original drawing is correct.**



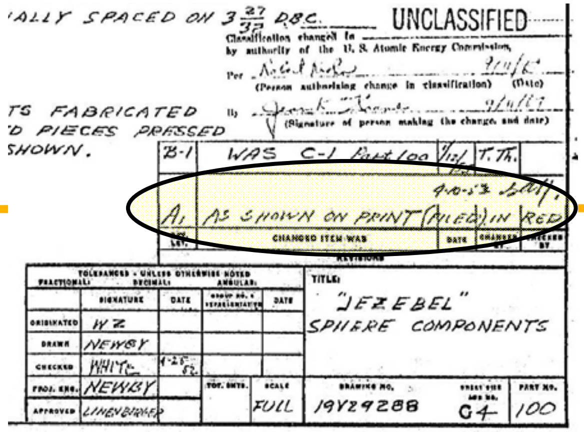
- Effect of this assumption in Rev. 3:

Conf.	Base C/E	0.871-in. C/E
A	1.00073	1.00026
B	1.00099	1.00052
C	1.00074	1.00025
D	1.00062	1.00012
Avg.	1.00077	1.00029

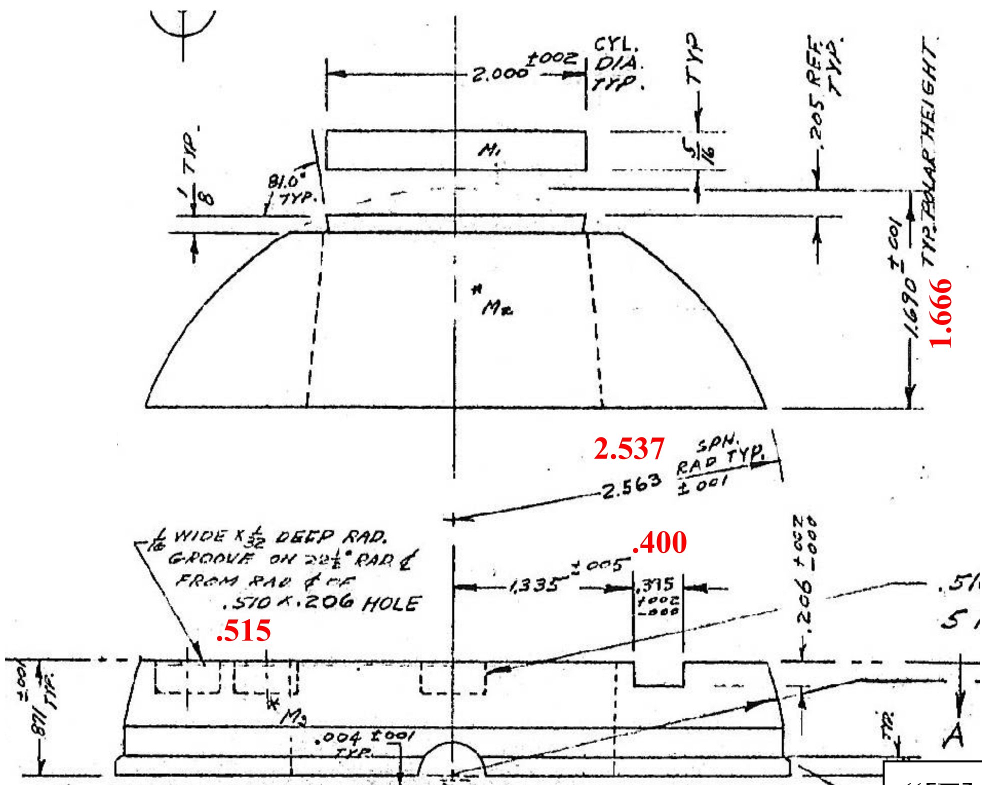
- The change in volume is +0.21% for upper M2 and +0.22% for lower M2.

- The difference of 48 pcm was included in the final uncertainty.

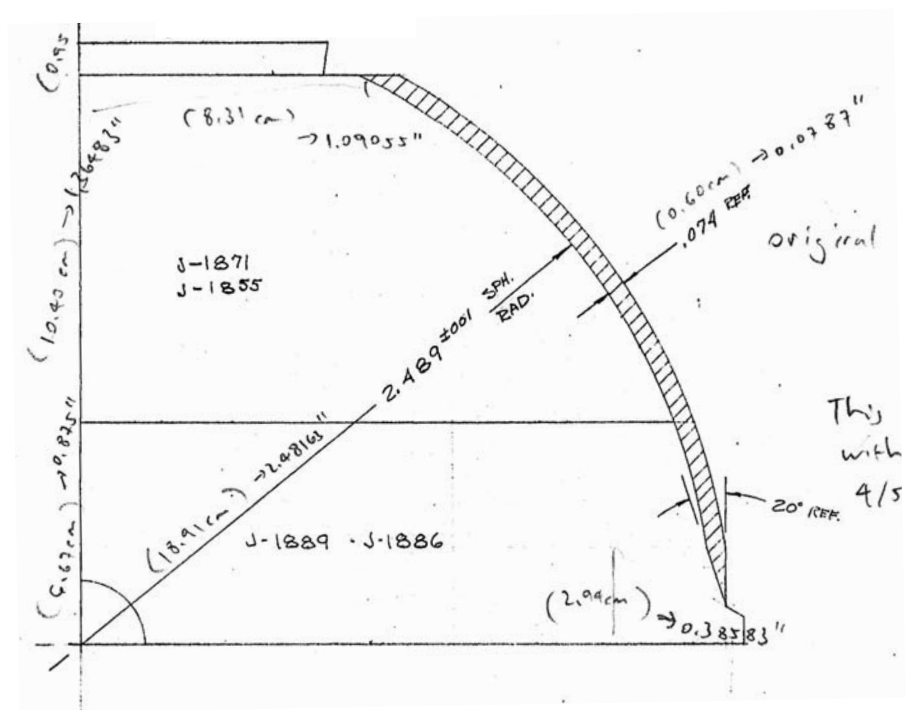
Ambiguity in the Dimensions ("Red Mark-up")



19Y29288 C4, revised April 1953



19Y29288 C6, Nov. 1954



"[F]our major parts were returned and 1.2 kg of material was removed by decreasing the ball radius .075"." – Group CMR-11 Monthly Report, Nov. 23, 1954

Assumption 3: “Red Mark-up” Change Was Not Implemented

- The original drawing (April 1952) has the change notation “As shown on print (filed) in red” (April 1953), and such a “red mark-up” drawing exists.

- But the “red mark-up” drawing is not consistent with documents describing the “remachining” of Nov. 1954 (radius of spherical parts was reduced 0.074 inch):
  - + The drawing (Nov. 1954) specifying the remachining (original outer radius matches April 1952 drawing).
  - + A CMR-11 report describing the remachining (the “four major parts were returned and 1.2 kg of material was removed by decreasing the ball radius .075"”).
    - – The density corresponding to the “red mark-up” drawing would be 24 g/cm^3.
    - – The density corresponding to the original drawing would be 15.66 g/cm^3.

- **Assumed original drawing is correct.**

  - Effect of this assumption in Rev. 3:

Conf.	Base C/E	“Red” C/E
A	1.00073	1.00170
B	1.00099	1.00207
C	1.00074	1.00174
D	1.00062	1.00166
Avg.	1.00077	1.00179

- The change in volume is –0.12% for upper M3, –0.44% for upper M2, –0.06% for lower M3, and –0.50% for lower M2.

- The difference of 102 pcm was not included in the final uncertainty but it is within the calculated uncertainty. It is a potential bias.

Detailed Model k_{eff} Uncertainties (Rev. 3)

Source	$\delta k_{eff}/k_{eff}$
Temperature Adjustment	±0.00003
Plutonium Mass, Dimensions, and Density	±0.00094
Imperfect Spherical Surfaces	Negligible
Lack of Planeness (Size of Gaps) Due to Nonuniform Ni	±0.00056
Lack of Planeness Due to Tilt	Negligible
Plutonium Isotopics	±0.00032
Decay of ^{241}Pu Before and During Experiments	Negligible
Gallium Content	±0.00001
Nickel Plating Thickness and Density	±0.00053
Assembly Machine and Hall Densities	±0.00002
Plutonium Impurities	±0.00002
Glory Hole Fill Position	Negligible
Button Placement	Negligible
Concrete Composition	Negligible
Absence of Framework and Other Assemblies	Negligible
Presence of α-Plutonium	Negligible
Miscellaneous Material Compositions	Negligible
Presence of Thermocouple	Negligible
Glory Hole Axis Offset	–0.00005
Button Dimensions	Negligible
Control Rod Position	±0.00013
Aluminum Spacer	±0.00017
Total	±0.00128

The total for the highlighted effects is ±0.000126.

Detailed Model k_{eff} Uncertainties (Rev. 3)

- Reactivity Impact of Systematic and Random Experimental Uncertainties

Source	$\delta k_{eff}/k_{eff}$
Systematic	±0.00128
Random	±0.00016
Total	±0.00129

- The final uncertainty is 65%-76% of that of the "original" one-dimensional model, for which no detailed uncertainty analysis was given.

One-Dimensional Model, PU-MET-FAST-001 Rev. 3

- Jezebel is a one-dimensional bare sphere critical plutonium benchmark.

  + Radius 6.3849 cm → 6.39157 cm
  + Density 15.61 g/cm^3 → 15.61 g/cm^3
  + Mass 17,020 ± 100 g Pu alloy → 17,073.2 ± 77 g
  + Material – gallium is separated into its isotopic constituents:

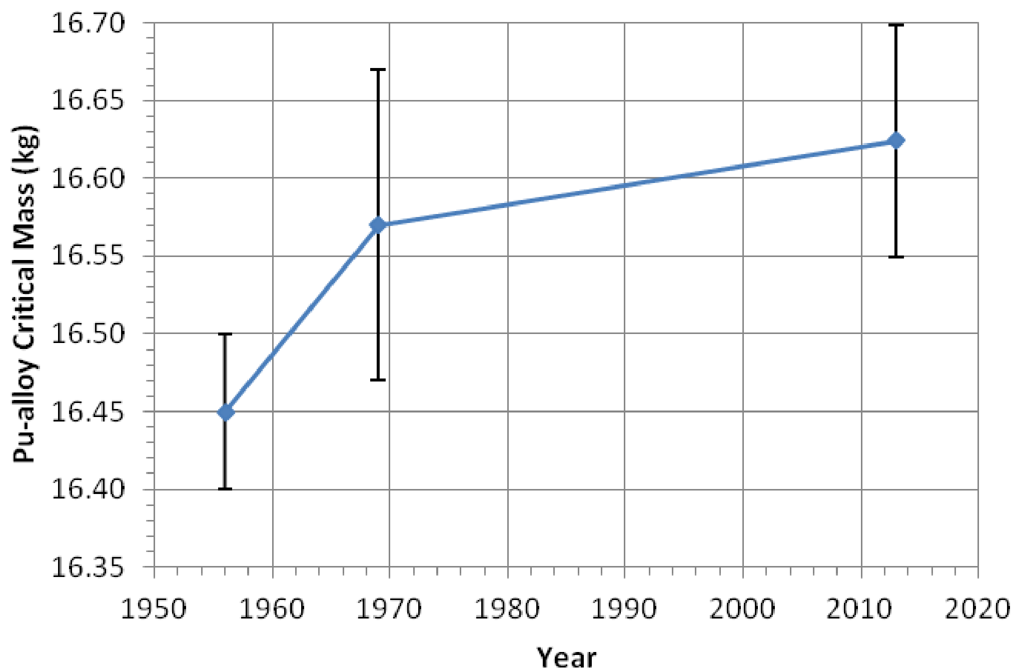
Nuclide	Atom Density (atoms/barn·cm)	Atom Fraction	Atom Fraction in Plutonium
^{69}Ga	8.3603×10^{-4}	2.0750×10^{-2}	N/A
^{71}Ga	5.3917×10^{-4}	1.3382×10^{-2}	N/A
^{239}Pu	3.7047×10^{-2}	9.1951×10^{-1}	0.952
^{240}Pu	1.7512×10^{-3}	4.3465×10^{-2}	0.045
^{241}Pu	1.1674×10^{-4}	2.8975×10^{-3}	0.003

- Benchmark k_{eff} 1.000 ± 0.002 → 1.00000 ± 0.00129.
  + ENDF/B-VII was tuned to the original one-dimensional Jezebel.
  + The average C/E of the four detailed models, using ENDF/B-VII, is 1.00077 ± 0.00129.
  + If the data were retuned to compute k_{eff} =1 for the new one-dimensional Jezebel, then it should compute C/E = 1 for the four detailed models.

So What Is the Critical Mass of a Bare Sphere of Plutonium?

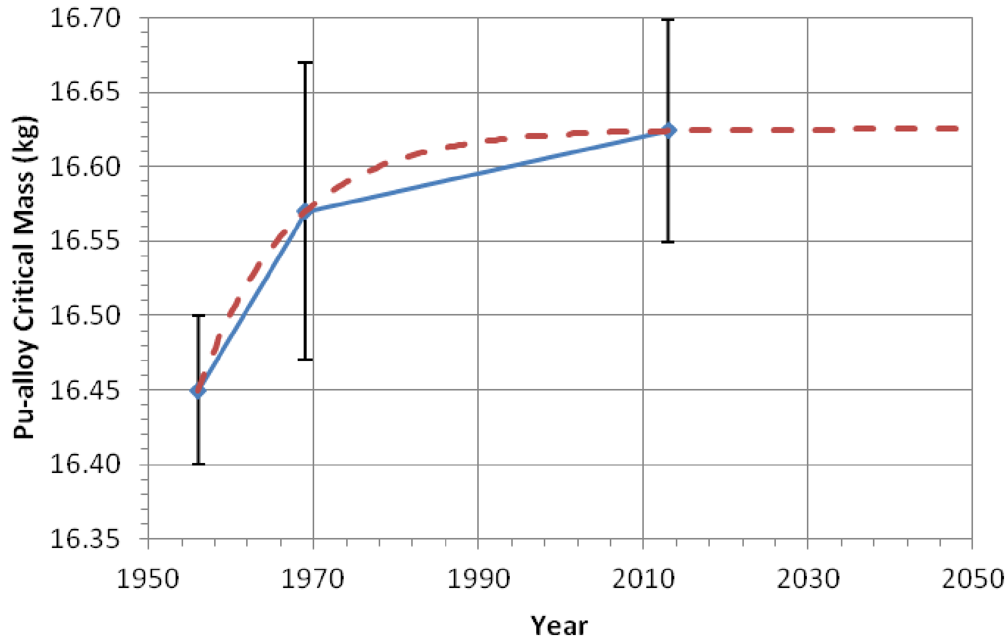
- The one-dimensional benchmark model uses 15.61 g/cm^3, determined in LA-4208.
- Using 15.82 g/cm^3, 1.02 wt.% Ga, Pu with 4.5 at.% ^{240}Pu:

Source	Year	Critical Mass of Pu-alloy (kg)
LA-2044	1956	16.45 ± 0.05
LA-4208	1969	16.57 ± 0.10
PU-MET-FAST-001 Rev. 3	2013	16.624 ± 0.075



The Critical Mass Has Leveled Off

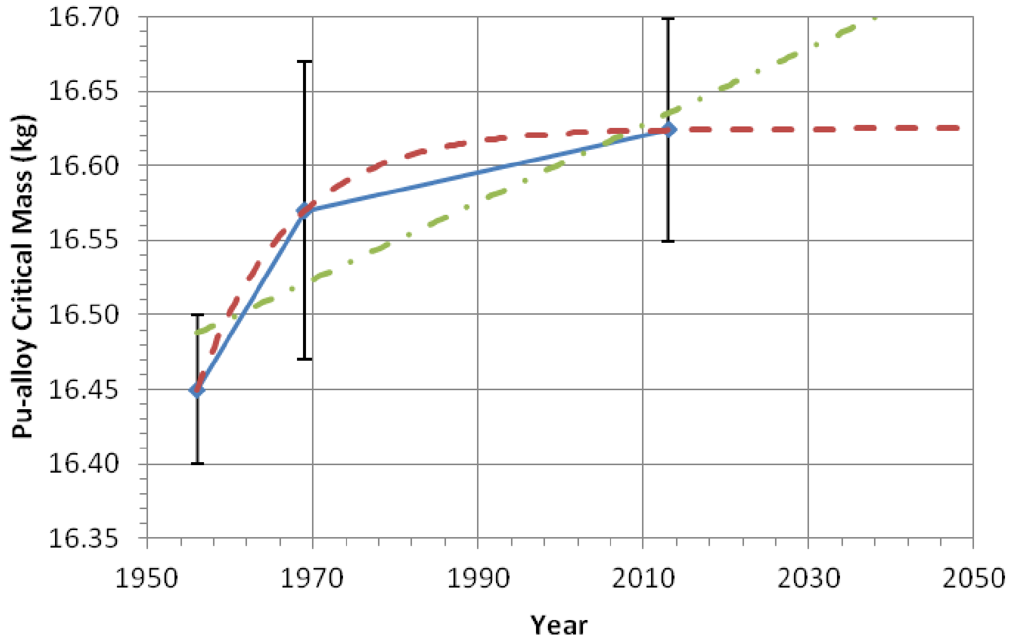
- $m_C = (0.1751\,\text{kg}) \times \left(1 - e^{-0.08894(y-1956)}\right) + 16.4500\,\text{kg}$, where y is the year AD.



- Asymptotic value is 16.625 kg.

…Or Has It?

- $m_C = (0.1751\,\text{kg}) \times \left(1 - e^{-0.08894(y-1956)}\right) + 16.4500\,\text{kg}$, where y is the year AD.
- $m_C = (0.0025876\,\text{kg/yr}) \times (y - 1956) + 16.4876\,\text{kg}$, where y is the year AD.



Summary and Conclusions

- We have reevaluated the classic Jezebel benchmark by modeling four actual experimental configurations as accurately as possible.

- In Rev. 3, when we did not know the part masses, the average k_{eff} C/E for the four detailed configurations was 1.00077. The uncertainty $\delta k_{eff}/k_{eff}$ was ±0.00129.
  + The reevaluated one-dimensional simplification (17.0732 kg ± 0.077 kg Pu-alloy) was statistically indistinguishable from the "original" one (LA-4208; 17.02 kg ± 0.6%).
  + Our ICSBEP report (PU-MET-FAST-001 Rev. 3) has been published (Sept. 2013).
  + In this evaluation, assumptions had to be made about the part masses and some of the dimensions. The uncertainties associated with these assumptions were accounted for and found to add only a small amount to the total uncertainty. If a mass statement for the original Jezebel parts from 1954 or 1955 can be found, this evaluation may be updated. It is unlikely, however, that the total k_{eff} uncertainty will be reduced below about ±0.00100.

- We have discovered part masses from 1954 and some part densities.
  + Part dimensions have to change dramatically to match the densities.
  + Testing various possibilities and working out the uncertainty will be a lot of work.

- It is not likely that we will learn which drawing was correct – but then the part volumes will be driven by the densities.

- The total uncertainty will not be below ±0.00100.

Acknowledgments

- Special thanks to Rev.3 co-author Roger Brewer (LANL) and external reviewer Ray Reed (Washington Safety Management Solutions).
- Thanks to Mark Chadwick (LANL) and Mark Williams (ORNL) for valuable discussions.
- Thanks to Brian Kiedrowski (LANL) for adjoint-based sensitivity calculations.
- Thanks to Billy Myers (LANL) for discovering the mass transfer receipt and density notes.